

Effects of an endoscope and rotation on peristaltic flow in a tube with long wavelength

G. A. Yahya^{a,b}, M. F. Sanaa^{a,b} and A. M. Abd-Alla^{c,d}

^a Physics Department Faculty of Science Taif University-K.S.A.

^b Physics Department Faculty of Science, Aswan University- Aswan 81528- Egypt.

^c Mathematics Department Faculty of Science Taif University-K.S.A.

^d Mathematics Department Faculty of Science, Sohage University- Sohage- Egypt.

E-mail: gamal102@yahoo.com

Abstract: The objective of this project is to study examines the problem of peristaltic flow in a tube with an endoscope subjected to rotation. The effect of an endoscope and rotation of the peristaltic flow of a Jeffrey fluid through the cylindrical cavity between concentric tubes is investigated. The analytical expressions for the pressure gradient, velocity, pressure rise, friction force on the inner and outer tubes and shear stress are obtained in the physical domain. Effect of the non-dimensional wave amplitude, the rotation, the ratio of relaxation to retardation time, the radius ratio and the non-dimensional volume flow is analyzed theoretically and computed numerically. Comparison is made with the results obtained in the presence of rotation and an endoscope.

Keywords: Peristaltic motion, an endoscope, rotation, Jeffrey fluid and Velocity profiles

1. Introduction

The study of peristaltic motion has gained considerable interest because of its extensive applications in urine transport from the kidney to bladder, transport of the spermatozoa in the ducts efferent's of the male reproductive tract, movement of the ovum in the fallopian tube, vasomotor of the small blood vessels, movement of the chime in gastrointestinal tract and so forth. Peristaltic pumping are found in many applications such as for the transport of slurries, sensitive or corrosive fluids, sanitary fluid, noxious fluids in the nuclear industry, to name but a few examples. The extensive

literature on the topic is available for viscous fluids. But the theory of non-Newtonian fluids has received great attention during the recent years, because the traditional viscous fluids cannot precisely describe the characteristics of many physiological fluids. The governing equations for such fluids are complicated and highly non-linear than the Navier--Stokes equations and present interesting challenges to physicists, computer scientists, mathematicians and modelers. Abd-Alla, et al. [1] investigated the effect of the rotation, magnetic field and initial stress on peristaltic motion of micropolar fluid.

Mahmoud, et al. [2] discussed the effect of the rotation on wave motion through cylindrical bore in a micropolar porous medium. Tripathi [3] studied a mathematical model for the peristaltic flow of chyme movement in the small intestine. Akbar, et al. [4] discussed the effects of heat and mass transfer on the peristaltic flow of hyperbolic tangent fluid in an annulus. Nadeem and Akbar [5] investigated the Influence of heat and chemical reactions on Walter's B fluid model for blood flow through a tapered artery. Nadeem and Akram [6] studied the peristaltic flow of a Williamson fluid in an asymmetric channel. Abd elmaboud [7] discussed the Influence of induced magnetic field on peristaltic flow in an annulus. Nadeem and Akram [8] have studied the peristaltic flow of a Jeffrey fluid in a rectangular duct. Ali, et al. [9] discussed the peristaltic flow of a Maxwell fluid in a channel with compliant walls. Walker and Shelley [10] studied the shape optimization of peristaltic pumping. Srinivas and Kothandapani [11] investigated the influence of heat and mass transfer on MHD peristaltic flow through a porous space with compliant walls. Pandey, et al. [12] discussed the peristaltic transport of multilayered power-law fluids with distinct viscosities: A mathematical model for intestinal flows. Jiménez-Lozano and Sen [13] studied the streamline topologies of two-dimensional peristaltic flow and their bifurcations. Nadeem and Akbar [14] discussed the effects of temperature dependent viscosity on peristaltic flow of a Jeffrey-six constant fluid in a non-uniform vertical tube". Pandey and Chaube [15] studied peristaltic flow of a micropolar fluid through a porous medium in the presence of an external magnetic field. Gad [16] discussed the effect of hall currents on interaction of pulsatile and peristaltic transport induced flows of a particle fluid suspension. Abd elmaboud and Mekheimer [17] studied the non-linear peristaltic transport of a second-order fluid through a porous medium. Maiti and Misra [18] studied the peristaltic flow of a fluid in a porous channel. Vajravelu et al [19] discussed the influence of heat transfer on peristaltic transport of a Jeffrey fluid in a vertical porous stratum. Hayat and Noreen [20] studied peristaltic transport of fourth grade fluid with heat transfer and induced magnetic field. Koshel [21] investigated the peristaltic flow of a Williamson fluid in an asymmetric channel. Yıldırım and Sezer [22] studied the

effects of partial slip on the peristaltic flow of a MHD Newtonian fluid in an asymmetric channel. The extensive literature on the topic is now available and we can only mention a few recent interesting investigations in refs. [26-34].

2. Formulation of the problem

We consider the motion of an incompressible viscous fluid through a porous medium in a two-dimensional tube (see Fig.) induced by sinusoidal wave trains propagation with constant speed c along the tube walls the wall is assumed to be a flexible membrane.

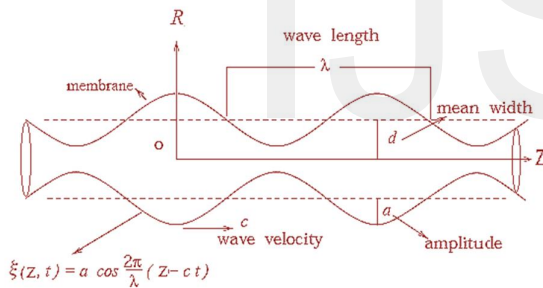


Figure (a): Geometry of cylindrical tube with peristaltic wave motion of wall.

The governing equations for the peristaltic motion of an incompressible micro polar fluid in the circular cylindrical coordinates (R, θ, Z) are

$$\frac{\partial \bar{U}}{\partial \bar{R}} + \frac{\partial \bar{W}}{\partial \bar{Z}} + \frac{\bar{U}}{\bar{R}} = 0 \quad (1)$$

$$\rho \left(\frac{\partial}{\partial \bar{t}} + \bar{U} \frac{\partial}{\partial \bar{R}} + \bar{W} \frac{\partial}{\partial \bar{Z}} \right) \bar{U} + 2\rho\Omega\bar{U} = -\frac{\partial \bar{P}}{\partial \bar{R}} + \frac{1}{\bar{R}} \frac{\partial}{\partial \bar{R}} (\bar{R}\bar{S}_{\bar{R}\bar{R}}) + \frac{\partial}{\partial \bar{Z}} (\bar{S}_{\bar{R}\bar{Z}}) - \frac{\bar{S}_{\bar{\theta}\bar{\theta}}}{\bar{R}} \quad (2)$$

$$\rho \left(\frac{\partial}{\partial \bar{t}} + \bar{U} \frac{\partial}{\partial \bar{R}} + \bar{W} \frac{\partial}{\partial \bar{Z}} \right) \bar{W} - 2\rho\Omega\bar{W} = -\frac{\partial \bar{P}}{\partial \bar{Z}} + \frac{1}{\bar{R}} \frac{\partial}{\partial \bar{R}} (\bar{R}\bar{S}_{\bar{R}\bar{Z}}) + \frac{\partial}{\partial \bar{Z}} (\bar{S}_{\bar{Z}\bar{Z}}) \quad (3)$$

$$\bar{P} = P - \frac{1}{2}\rho\Omega^2 \frac{d\bar{X}^2}{d\bar{r}} \quad (4)$$

In which \bar{U} and \bar{W} are the velocity components in the \bar{R} and \bar{Z} -directions, respectively, $\Omega = \Omega e_\theta$, e_θ is the unit vector, $\Omega = (0, \Omega, 0)$ is the rotation vector, X is given by $X^2 = R^2 + Z^2$, \bar{P} is the modified pressure, the equations of motion in the rotating frame has two additional terms $\rho\Omega \times (\Omega \times X)$ is the centrifugal force, $2\rho(\Omega \times V)$ is the Coriolis force, $\bar{X} = (R, 0, Z)$ and $V = (U, 0, W)$ is the velocity vector.

The constitutive equation for the extra stress tensor \bar{S} for Jeffrey fluid is

$$\bar{S} = \frac{\mu}{1+\lambda_1} (\bar{\gamma} + \lambda_2 \dot{\bar{\gamma}}) \quad (5)$$

where, μ is the dynamic viscosity, $\bar{\gamma}$ is the shear rate, λ_1 is the ratio of relaxation to retardation times, λ_2 is the retardation time and dots over the quantities denote differentiation. In the fixed coordinates (\bar{R}, \bar{Z}) , the flow between the two tubes is unsteady. It becomes steady in a wave frame (\bar{r}, \bar{z}) moving with the same speed as wave in

the \bar{Z} -direction. The transformations between the two frames are:

$$\bar{r} = \bar{R} \quad , \quad \bar{z} = \bar{Z} - c\bar{t} \quad (6)$$

$$, \quad \bar{u} = \bar{U} \quad \text{and} \quad \bar{w} = \bar{W} - c \quad (7)$$

where \bar{u} and \bar{w} are the velocities in the wave frame.

The appropriate boundary conditions in the frame are of the following form:

$$\begin{aligned} \bar{w} &= -c, \quad \bar{u} = 0 \text{ at } \bar{r} = \bar{r}_1 \quad \text{and} \\ \bar{w} &= -c, \quad \text{at } \bar{r} = \bar{r}_2 + b \sin \frac{2\pi}{\lambda} \bar{z} \end{aligned} \quad (8)$$

Now we introduce the non-dimensional parameters and variables as follows:

$$\begin{aligned} r &= \frac{\bar{r}}{a_2} \quad , \quad z = \frac{\bar{z}}{\lambda} \quad , \quad u = \frac{\lambda \bar{u}}{a_2 c} \\ P &= \frac{a_2^2 \bar{P}}{c \lambda \mu} \\ r_1 &= \frac{\bar{r}_1}{a_2} = \epsilon < 1, \quad w = \frac{\bar{w}}{c} \quad , \quad S = \frac{a_2 \bar{S}}{\mu c} \\ r_2 &= \frac{\bar{r}_2}{a_2} = 1 + \phi \sin(2\pi z) \end{aligned} \quad (9)$$

Where \bar{u} and \bar{w} are the velocities in wave frame

In view of the equations (1) to (9) we have:

$$\begin{aligned} Re \delta^3 \left(u \frac{\partial}{\partial r} + w \frac{\partial}{\partial z} - \frac{2\Omega\lambda}{c} \right) u &= -\frac{\partial P}{\partial r} + \\ \frac{\delta}{r} \frac{\partial}{\partial r} (r S_{rr}) + \delta^2 \frac{\partial}{\partial z} (S_{rz}) - \delta \frac{S_{\theta\theta}}{r} \end{aligned} \quad (10)$$

$$\begin{aligned} Re \delta \left(u \frac{\partial}{\partial r} + w \frac{\partial}{\partial z} \right) w - \frac{2\Omega a_2^2 \rho}{\mu} w &= -\frac{\partial P}{\partial z} + \\ \frac{1}{r} \frac{\partial}{\partial r} (r S_{rz}) + \delta \frac{\partial}{\partial z} (S_{zz}) \end{aligned} \quad (11)$$

$$\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} + \frac{u}{r} = 0 \quad (12)$$

$$\begin{aligned} w &= -1, \quad u = 0 \text{ at } r = r_1 = \epsilon \\ w &= -1, \quad \text{at } r = r_2 = 1 + \phi \sin(2\pi z) \end{aligned} \quad (13)$$

Where ϵ is the radius ratio, $\phi (= \frac{b}{a_2} < 1)$ is the wave amplitude and

$$\begin{aligned} S_{rr} &= \frac{2\delta}{1+\lambda_1} \left[1 + \frac{\lambda_2 c \delta}{a_2} \left(u \frac{\partial}{\partial r} + w \frac{\partial}{\partial z} \right) \right] \frac{\partial u}{\partial r} \\ S_{rz} &= \frac{1}{1+\lambda_1} \left[1 + \frac{\lambda_2 c \delta}{a_2} \left(u \frac{\partial}{\partial r} + w \frac{\partial}{\partial z} \right) \right] \left(\frac{\partial w}{\partial r} + \delta^2 \frac{\partial u}{\partial z} \right) \\ S_{\theta\theta} &= \frac{2\delta}{1+\lambda_1} \left[1 + \frac{\lambda_2 c \delta}{a_2} \left(u \frac{\partial}{\partial r} + w \frac{\partial}{\partial z} \right) \right] \frac{u}{r} \\ S_{zz} &= \frac{2\delta}{1+\lambda_1} \left[1 + \frac{\lambda_2 c \delta}{a_2} \left(u \frac{\partial}{\partial r} + w \frac{\partial}{\partial z} \right) \right] \frac{\partial w}{\partial z} \end{aligned} \quad (14)$$

The wave number δ and Reynolds number Re are

$$\begin{aligned} \delta &= \frac{a_2}{\lambda} \\ \text{and} \quad Re &= \frac{\rho c a_2}{\mu} \end{aligned} \quad (15)$$

3. Solution of the problem

Using long wavelength approximation and neglecting the wave number along with low Reynolds number, one can find from Eqs. (10) and (11) yield

$$-\frac{\partial P}{\partial r} = 0 \quad (16)$$

$$-\frac{2\Omega a_2^2 \rho}{\mu} w = -\frac{\partial P}{\partial z} + \frac{\partial S_{rz}}{\partial r} + \frac{S_{rz}}{r} \quad (17)$$

By substituting for S_{rz} in the above equation, multiplying by rdr and integrated the resulting equation, we get:

$$\frac{\partial w}{\partial r} = (\lambda_1 + 1) \left[\frac{c_1}{r} - \frac{\Omega a_2^2 \rho r w}{\mu} + \frac{r}{2} \frac{dP}{dz} \right] \quad (18)$$

By integration again, we get

$$w = \frac{c_2}{\left(\frac{\Omega a_2^2 \rho r^2}{2\mu} + \frac{1}{(\lambda_1+1)}\right)} + \frac{c_1 \ln r}{\left(\frac{\Omega a_2^2 \rho r^2}{2\mu} + \frac{1}{(\lambda_1+1)}\right)} + \frac{r^2}{4\left(\frac{\Omega a_2^2 \rho r^2}{2\mu} + \frac{1}{(\lambda_1+1)}\right)} \frac{dp}{dz} \quad (19)$$

Where c_1 and c_2 are the integration constants

The appropriate boundary conditions in the wave frame are the following form:

$$\begin{aligned} \bar{w} &= -c, & \bar{u} &= 0 \text{ at } r = r_1 = \varepsilon \\ \bar{w} &= -c, & & \text{ at } r = r_2 + b \sin \frac{2\pi}{\lambda} z \end{aligned} \quad (20)$$

Applying the boundary conditions (20) on the solution (19), we get the constants c_1 and c_2 as:

$$\begin{aligned} c_1 &= \frac{\beta_1 - \beta_2 + \frac{1}{4} \frac{dp}{dz} (r_1^2 - r_2^2)}{(\ln r_2 - \ln r_1)} \\ c_2 &= \frac{\ln r_1 \left(\frac{4\beta_2 + r_2^2 \frac{dp}{dz}}{4} \right) - \ln r_2 \left(\frac{4\beta_1 + r_1^2 \frac{dp}{dz}}{4} \right)}{(\ln r_2 - \ln r_1)} \end{aligned} \quad (21)$$

where

$$\begin{aligned} \beta_1 &= \left(\frac{\Omega a_2^2 \rho r_1^2}{2\mu} + \frac{1}{(\lambda_1+1)} \right) \\ \text{and } \beta_2 &= \left(\frac{\Omega a_2^2 \rho r_2^2}{2\mu} + \frac{1}{(\lambda_1+1)} \right) \end{aligned} \quad (22)$$

From equations (19) and (12) subject the boundary conditions (20) we get the radial displacement as:

$$u = -\frac{1}{8A^2} \frac{d^2 P}{dz^2} \left(\frac{1}{r_1} - \frac{1}{r} \right) [A(r_1^2 - r^2) - B(\log(B - Ar_1^2) + \log(B - Ar^2))] \quad (23)$$

Where

$$A = \frac{\Omega a_2^2 \rho}{2\mu}$$

$$\text{and } B = \frac{1}{(\lambda_1+1)} \quad (24)$$

The non-dimensional expressions for pressure rise (ΔP_λ) and frictional forces (F_λ^i), (F_λ^o) on the inner and outer tube are respectively given by:

$$\Delta P_\lambda = \int_0^1 \left(\frac{dp}{dz} \right) dz \quad (25)$$

$$F_\lambda^i = \int_0^1 r_1^2 \left(-\frac{dp}{dz} \right) dz \quad (26)$$

$$F_\lambda^o = \int_0^1 r_2^2 \left(-\frac{dp}{dz} \right) dz \quad (27)$$

4. Numerical results and discussion

The values of the pressure gradient $\frac{dP}{dz}$ radial velocity u axial velocity w pressure rise ΔP_λ , friction force on the inner tube F_λ^i and friction force on the outer tube F_λ^o have been discussed by assigning numerical values to the parameter encountered in the problem in which the numerical results are displayed with the graphic illustrations. The variations are shown in Figs. 1-12

Figs. (1)-(3) show that the variations of the values of pressure gradient $\frac{dP}{dz}$, axial velocity w and pressure rise ΔP_λ with respect to the radial r which it decreases with increasing of the radial r in the whole range of the r -axis for different values of the rotation Ω , while it decreases with increasing of rotation.

Fig. (4) shows that the variations of the values of radial velocity u with respect to the radial r which it has oscillatory behavior in the whole range of the r -axis, while it increases with increasing of rotation Ω .

Figs. (5)-(6) show that the variations of the value of friction force on the inner tube F_{λ}^i and the value of friction force on the outer tube F_{λ}^o with respect radial r , which it decreases with increasing of the radial r and rotation Ω .

Figs. (7)-(8) show that the variations of the value of pressure gradient $\frac{dP}{dz}$ and axial velocity w with respect to the radial r which it decreases with increasing of the radial r in the whole range of the r -axis for different values of the radius ratio ε , while the axial velocity has oscillatory in the whole range of r , as well it increases with increasing of the radius ratio.

Fig. (9) shows that the variations of the value of pressure rise ΔP_{λ} with respect to the radial r which it decreases with increasing of the radius ratio ε and radial r .

Fig. (10) shows that the variations of the values of radial velocity u with respect to the radial r which it has oscillatory behavior in the whole range of the r -axis, while it increases with increasing of radius ratio ε .

Figs. (11)-(12) show that the variations of the value of friction force on the inner tube F_{λ}^i and the value of friction force on the outer tube F_{λ}^o with respect radial r , which it decreases with increasing of the radial r and radius ratio ε .

5. Conclusion

Due to the complicated nature of the governing equations of the pertinent field equations governing the peristaltic flow, the work done in this field is unfortunately limited in number. The method used in this study provides a quite successful in dealing with such problems. This method gives exact solutions in the peristaltic flow without any assumed restrictions on the actual physical quantities that appear in the governing equations of the problem considered. Important phenomena are observed in all these computations:

- It was found that for large values of the rotation and radius, the case is quite different when we consider small value of rotation taking into consideration that the solutions obtained in the context of the peristaltic flow of a Jeffrey fluid.
- A comparison with previous studies, we find that our results have been obtained

without an endoscope coincides with the results obtained by Siddique and Schehawey [35] for Newtonian fluid. Also, this result agrees with those of Shapiro et al. [36] and Shukla et al. [37] when the rotation Ω asymptotes to zero

•The results presented in this paper should prove useful for researchers in scientific and engineering, as well as for those working on the development of a theory of hyperbolic propagation of hyperbolic fluid mechanics. The non-dimensional wave amplitude, magnetic field, ratio of relaxation to retardation time, radius ratio, non-dimensional volume flow, rotation, density, and the radius exchange with the endoscope influence and operations. Study of the phenomenon of the non-dimensional wave amplitude, magnetic field, ratio of relaxation to retardation time, radius ratio, non-dimensional volume flow, rotation, density, and the radius are also used to improve the conditions of peristaltic flow.

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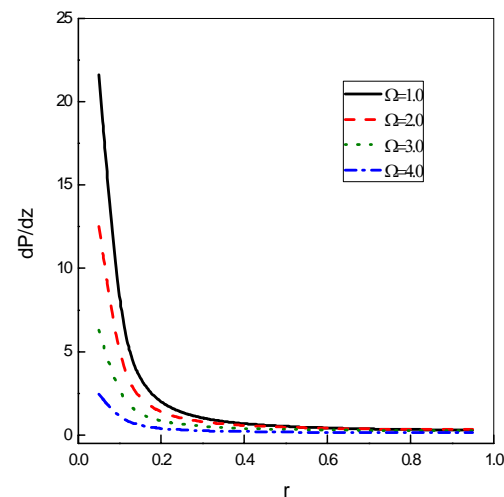


Fig. 1 Non-dimensional pressure gradient $\frac{dP}{dz}$ versus the radial r for different values of Ω .

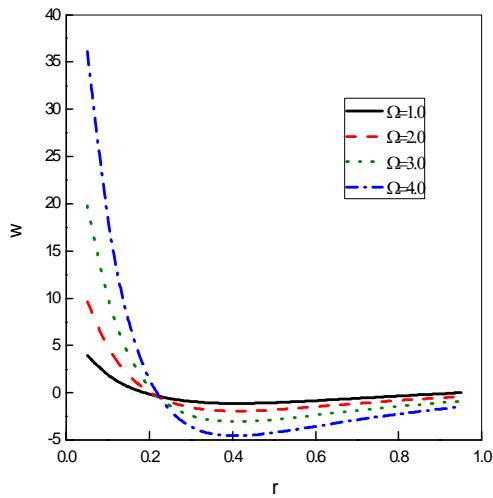


Fig. 3 Non-dimensional pressure rise ΔP_λ versus the radial r for different values of Ω .

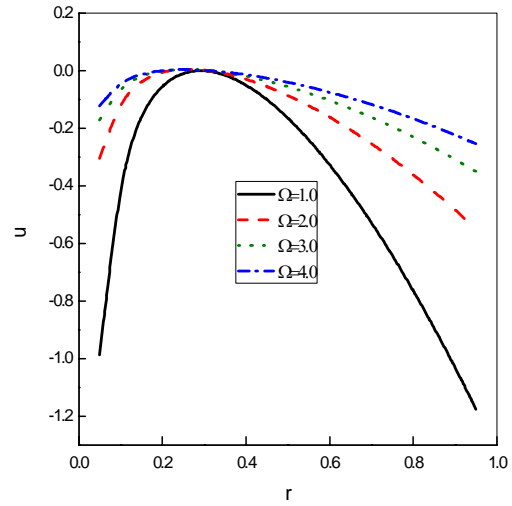


Fig. 2 Non-dimensional axial velocity w versus the radial r for different values of Ω .

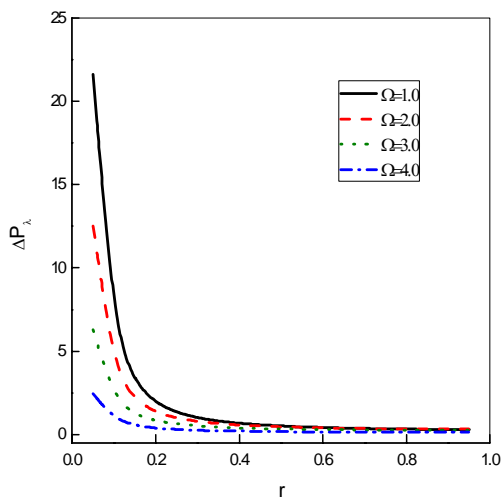


Fig. 4 Non-dimensional radial velocity u versus the radial r for different values of Ω .

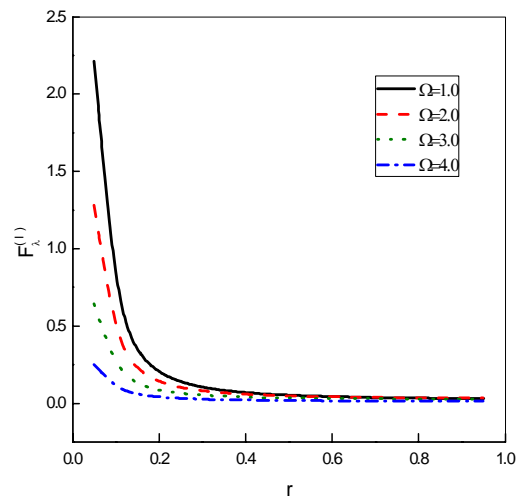


Fig. 5 Non-dimensional friction force on the inner tube F_{λ}^i versus the radial r for different values of Ω .

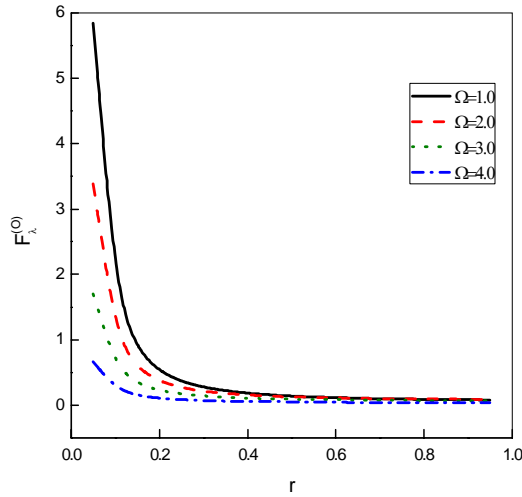


Fig. 7 Non-dimensional pressure gradient $\frac{dP}{dz}$ versus the radial r for different values of ε .

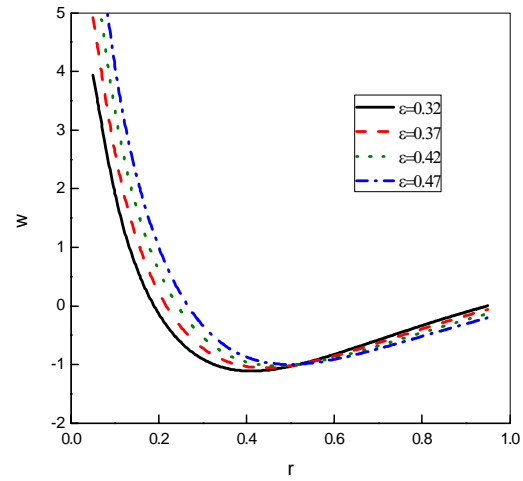


Fig. 6 Non-dimensional friction force on the outer tube F_{λ}^o versus the radial r for different values of Ω .

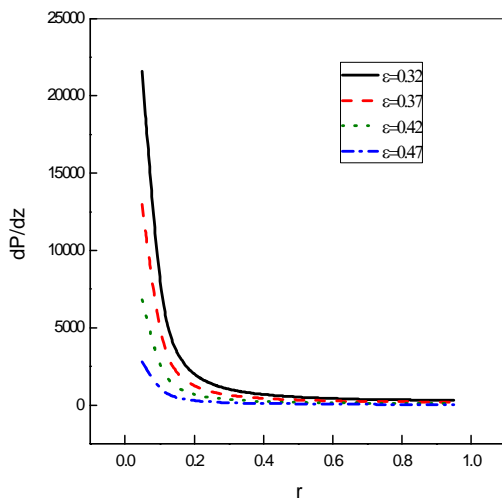


Fig. 8 Non-dimensional axial velocity w versus the radial r for different values of ε .

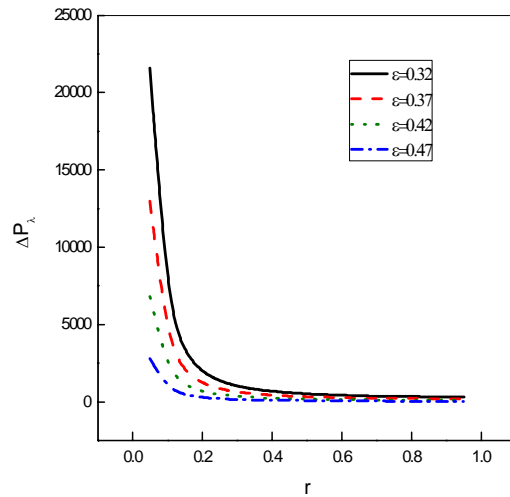


Fig. 9 Non-dimensional pressure rise ΔP_λ versus the radial r for different values of ε .

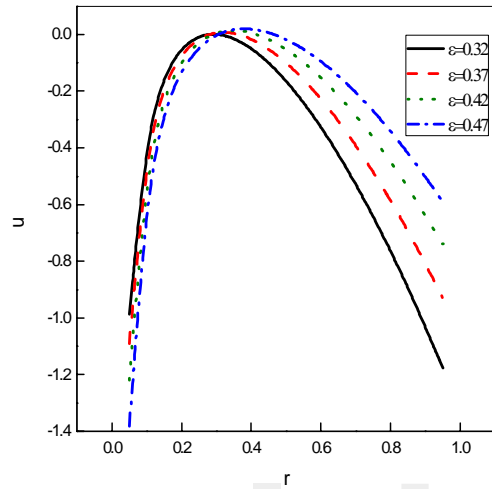


Fig. 11 Non-dimensional friction force on the inner tube F_λ^i versus the radial r for different values of ε .

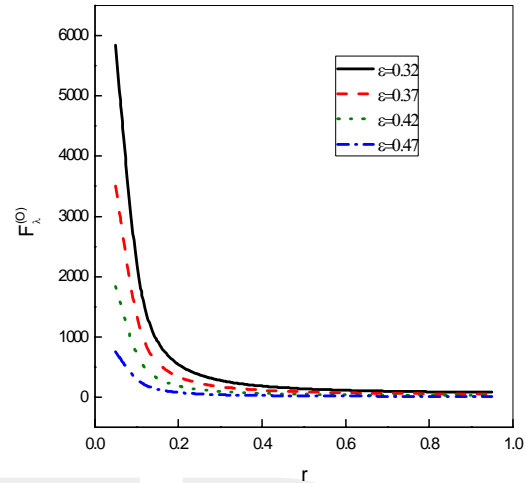


Fig. 10 Non-dimensional radial velocity u versus the radial r for different values of ε .

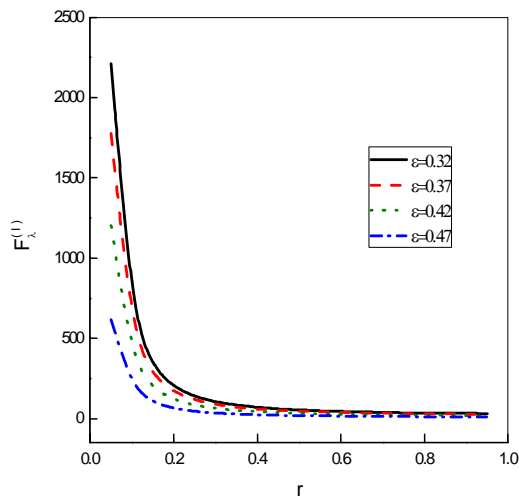


Fig. 12 Non-dimensional friction force on the outer tube F_λ^o versus the radial r for different values of ε .